Unequal admissions in England: choice inequality and strategies in secondary school admissions

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Abstract

Data from the English secondary school admissions cycle reveals a substantial ethnic gap in the proportion accessing their chosen schools. Children from ethnic minority groups are, on average, 17% less likely to be admitted into their top-ranked secondary school, compared to white children. This gap is large even after controlling for socio-economic status and prior attainment. However, constraints on the length of preference lists mean that stated preferences may not be truthful, as parents may have incentives to strategise by excluding preferred schools. By accounting for strategic stated preferences, this paper recovers parametric estimates of preference parameters and the degree of strategic selection. The corrected revealed preference estimates are used to estimate welfare comparisons between demographic groups. The analysis reveals that both underlying preferences and strategising behaviour exhibit substantial heterogeneity by demographic. When comparing welfare, differences in chances of admission between white and ethnic minority families create small but significant and consistent differences in welfare.

1 Introduction

England has a well-established quasi-market for state-funded secondary schools, in which parents have a right to express their preferences over schools, while parental choice is constrained by capacity at demanded schools, and popular schools are usually oversubscribed. In spite of this, headline figures of parental satisfaction usually report that the vast majority (around 90%) of families achieve their first choice of secondary school.

Furthermore, when these headline proportions are disaggregated by location, socio-economic status and ethnicity, substantial heterogeneity is revealed, with minority ethnicities in particular much less likely to achieve their first choice school. These strong empirical patterns in admissions, which have only recently been revealed by the availability of disaggregated data, require an explanation that takes into account geography, admissions priority rules, heterogeneity of preferences, and the possibility of strategic choice.

As welfare indicators, these statistics rely on strong assumptions of truthful preference reporting. The admissions system is intended to encourage truthful reporting of parents' preferences. However, constraints on the length of preference lists, together with misunderstandings about the mechanism, introduce the potential for strategic reporting (Haeringer and Klijn 2009). This not only causes biases in standard estimates of parental satisfaction, but also makes interpreting choice behaviour using reported preferences difficult. Perhaps most importantly, it adds to the psychological and cognitive burden of parents negotiating a decision that will have long-lasting effects on their children's education.

In this paper, I use a simple approximation to a structural model of choice under uncertainty, together with a data-driven identification strategy, to address strategic reporting, and provide estimates of parental preferences, welfare and access to schools for different demographic groups in different locations. The paper uses the universe of almost 500,000 admission decisions from the 2013 secondary school admissions cycle in England, to predict the probability of admission for any child at any school. This requires taking account of the non-random selection of the sample, and the existence of unobservables that strongly affect admission, but which are revealed by parents' choice behaviour. To account for this, I use the rich information available in stated rank-order lists to correct the estimated structural admissions function for selection on unobservables.

Using a random sample from the rank-order lists from the following admissions cycle (2014), I then estimate a model that approximates the strategic list selection under uncertainty about chances of admission. Estimates of the probability of admission are imputed to each school in each family's choice set, using the corrected structural admission function estimated on the previous year's data. This attenuates the bias arising from strategic choice and allows the estimation of willingness to travel for observable aspects of school quality.

Finally, the paper presents estimates of the gap in expected log utility, between parents' true preferred school, and the best school that they can achieve given capacity constraints, and the school that they actually achieve. These comparisons provide evidence on the relative contributions of admissions constraints, geography and sub-optimal strategies to differences in parental satisfaction between groups.

The analysis supports the findings in previous literature (cf. Deming et al. 2014; Burgess et al. 2015) that preferences, measured by marginal rates of substitution between school proximity and other observables, are heterogeneous across demographic groups. In contrast to previous work, the results emphasise heterogeneity by ethnicity rather than socio-economic status, although both exist. Ethnic minority families have a higher willingness to travel for observable aspects of school quality relating to academic performance, but they appear to place less emphasis on peer-related measures such as the proportion of children eligible for Pupil Premium.

The analysis also reveals both significant levels of strategic reporting, differing by ethnicity, income and ability, and small differences in the admissions treatment of different demographic groups, leading to a gap in welfare that increases when accounting for strategy. In particular, I find that ethnic minority families rank more schools, on average, than white parents, but they take less account of chances of admission in choosing schools to rank. The results of the welfare analysis suggest that, before taking capacity constraints into account, there is little to no difference in the quality of choice sets for different demographic groups, implying that geography plays only a small role in welfare differences. However, taking into account admissions probability, a gap emerges in welfare between white and ethnic minority families, and between non-Pupil Premium and

Pupil Premium families. This gap increases when comparing the welfare of optimal strategies to the welfare actually achieved, implying that differences in strategy exacerbate choice inequalities between demographic groups.

This work is related to a growing literature combining structural empirical modelling of strategic choice with topics from mechanism design (Calsamiglia, Haeringer, and Klijn 2010; Agarwal and Somaini 2018; Calsamiglia, Fu, and Güell 2014 ; Fack, Grenet, and He 2015; Ajayi and Sidibé 2016; Abdulkadiroğlu, Agarwal, and Pathak 2017; Luflade 2018). The contribution of this paper is to analyse the variation in strategy across socio-economic, ability and ethnic groups, and relate these differences in strategic choice behaviour to systemic inequality in access to good schools.

2 Setting and Data

2.1 English secondary school admissions

In England, children transition from primary school to secondary school at age 11. To apply for a place at a state secondary school in England, parents submit a ranking of their preferred schools. They can list between 3 and 6 schools depending on the Local Authority (LA). Parents must apply within their own LA, but may include any school within or outside their own LA on their list. Places are allocated using a deferred-acceptance (DA) algorithm, co-ordinated by each LA or collaborations between neighbouring LAs. For oversubscribed schools, allocation of places is prioritised based on a set of criteria which depends on the school, typically including: whether the child is in the care of the local authority ("looked-after" children); whether the child has an older sibling at the school; and, finally, the distance from the school. In some schools, religious worship or baptism forms the main criterion. In a small proportion of state schools, priority is given to those displaying an aptitude at a particular subject or a range of subjects. Finally, there are a small number of schools which fill all of their places based on performance in an exam, known as grammar schools. If a pupil cannot be allocated to any of their listed preferred schools, they are assigned to a school with spare capacity. In spite of this, many parents list fewer than the maximum allowed number of schools.

Schools can be categorised into six broad groups, according to their admissions policy and ethos.

Community schools The first group comprises those schools owned and controlled by LAs, comprising 20% of secondary school places in 2014. These schools generally have simple admissions criteria, prioritising siblings, "looked after" children and those living within a designated zone, with straight-line distance used as a tie-breaker. Until recently the largest group, since 2010 many formerly community schools have been converted into state-funded private schools called "academies".

Non-faith academies This second group is now the largest group, enrolling 57% of state-funded secondary school children. These schools have some autonomy to set their own admissions criteria although within the strict guidelines set by the government. Many academies have similar admissions criteria to community schools.

Roman Catholic schools 11% of secondary school children are enrolled in Roman Catholic schools, making this the largest faith school denomination in secondary schooling. RC schools usually select up to 100% of their intake on religious grounds. At many schools proof of baptism is sufficient, although at the more popular schools proof of regular church attendance may be required. Some schools reserve a proportion of places for children of other faiths/no faith.

Church of England schools The second largest providers of denominational secondary schooling are Church of England (C of E) schools (7% of places). A majority of C of E schools also require proof of religious worship for some or all places. The admissions criteria of CofE schools are more heterogeneous, and support for religious selection less unanimous in the Anglican sector than in the Roman Catholic sector.

Other faith schools There are a very small number (less than 1%) of schools with religious denomination other than Christian. These schools often operate faith-based admissions criteria for some or all places.

Grammar schools There are 162 academically-selective grammar schools (enrolling 5% of children) – the remainder of a much larger system of academic selection that existed prior to the 1970's. To obtain a place at a grammar school children are required to sit an academic exam. In some Local Authorities all 11-year old children sit a common exam, whereas in others only children who wish to apply to a school sit the school's exam.

Strict government guidelines regulate the admissions criteria that schools are permitted to use. Some unlawful criteria include: interviews or other face-to-face contact; rank-order (eg. first-preferences-first); parental contributions or volunteering. In addition, new schools have more stringent restrictions on religious and academic selection than existing schools. All schools, except community schools, receive a list of applications (without information on the preference order) and rank them before sending the ranking back to the LA for the matching to be computed. For community schools the LA computes the ranking.

It is worth noting that, in contrast to many other countries operating centralised school admissions, lotteries are almost unheard of as a priority tie-breaking mechanism. The final tie-breaker for the majority of community, academy and religious schools is straight-line distance between the school and the pupil's home address. This reliance on geographical priorities means that admission to schools is deterministic and depends only on priority, distance from the school, and the actions of other parents. However, uncertainty about the actions of other parents, combined with possible uncertainty about one's own priority, introduces subjective uncertainty from the point of view of the decision-maker.

2.2 Strategic reporting

There are two aspects of this system that may lead to strategic reporting. First, although, under student-proposing DA, truth-telling is a weakly dominant strategy (reference) this is not necessarily the case when preference lists are short and parents are forced to truncate their preference reports (Haeringer and Klijn 2009). We present evidence that in many Local Authorities, 3 to 6 slots are not sufficient and therefore parents are incentivised to consider their chances of admission when reporting a subset of their true preferences, to avoid being assigned to a school with spare capacity. The second aspect is the complexity of the matching mechanism, and uncertainty in admissions probabilities, that leads to erroneous beliefs about optimal strategies. It is suspected that misunderstanding of the rules of the matching mechanism is widespread.

In the presence of strategic reporting, the observed proportion of parents achieving their reported first-choice school is not a good indicator of welfare. In addition, differences in this measure by ethnicity, socio-economic status or other demographic variables may reflect differences in strategic behaviour, rather than real differences in welfare. Indeed, raw estimates of parental satisfaction, disaggregated by ethnicity, Pupil Premium eligibility (a proxy for income), prior attainment and location, reveal demographic and geographic heterogeneity. A pattern that stands out is the lower proportion of ethnic minority families achieving their first-choice school. To what extent does this empirical regularity reflect differences in parental satisfaction (welfare), and to what extent does it reflect differences in strategic behaviour?

2.3 Data and sampling

In 2014 the English Department for Education began to collect individual reported preference lists from all Local Authorities in England. We exploit the first two cohorts of secondary school entrants in the 2014 and 2015 admissions cycles. Each cohort contains choices for around 500,000 children who are in their final year of primary school (aged 10-11), linked to their home postcode, ethnicity, gender, Pupil Premium status (a proxy for low income), and primary school test scores from the National Pupil Database (NPD). From the home postcode we can calculate their location and home-school distances. The NPD also provides detailed information on schools, including location, test scores, demographic composition, governance and religious denomination. We also obtain current and historical government inspections (Ofsted) scores from the online schools database, Edubase.

We split pupils into 12 demographic groups defined by the following categorisations: white/minority ethnicity; eligible/not eligible for Pupil Premium (PP); Top/middle/bottom tercile of primary school (KS2) attainment.

2.4 Empirical evidence on ethnicity and admissions

As discussed above, the detailed break-down of parental satisfaction statistics by location, ethnicity, ability and other factors only became possible when the Department for Education released individualised preference records. Figure 1 reveals that the difference in the proportion of children from minority ethnic groups achieving their first choice is much larger than expected. Conversely, the difference in the percentage achieving first choice of those families eligible for Pupil Premium is more modest.

Table 4 shows that in London, Birmingham and Manchester, black, South Asian and other minority ethnic families rank more schools on average than white parents. Understanding the reasons for these differences, and the extent to which they evidence unequal chances of admission to schools due to geography or other factors, or diversity in school-choice strategies, is the focus of this study.

3 Empirical methods

I leverage a structural model of admission probability, that corrects for selection-on-unobservables in the observation of school admission decisions and directly estimates the sharp admission boundaries induced

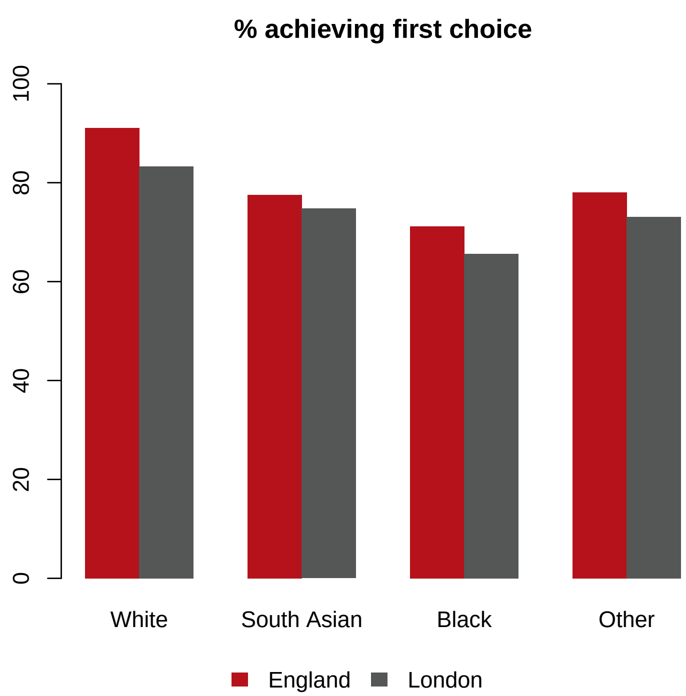

Figure 1: *Proportions obtaining a place at their first-choice school in 2014, by ethnic group, for London and England.*

by straight-line distance tie-breakers. To avoid simultaneity problems, two consecutive cohorts of students are used. The probabilities of admission, estimated using the first cohort, are imputed to the second cohort, averaging over the distribution of unobservables in the population.

3.1 Estimating admission probabilities

To model strategic choice under uncertainty about admission probabilities, the usual approach is to undertake a two-stage analysis, where the first stage involves modelling the admission probabilities themselves, or beliefs about them. Several authors have addressed this empirical problem. The most common approach to constructing admission probabilities is to take account of uncertainty in the admission thresholds, or cutoffs, at each school caused by the unknown distribution of other applicants and their preferences or strategies. In some applications authors also take account of uncertainty about the priority rule, the realised priorities, or both.

In order to estimate a portfolio-choice model for exam-based admission to Ghanian secondary schools, Ajayi and Sidibé (2016) model subjective probabilities of admission derived from uncertainty about both personal exam performance, and score thresholds (cutoffs) of schools. Decision makers infer exam cutoffs with error by observing cutoffs in previous years at the same school. Luflade (2018) also models the distribution of exam admissions cutoffs as inferred from previous years' observations, for a model of university choice under constrained lists in Tunisia.

Fack, Grenet, and He (2015) obtain data on students' priority scores and schools' cutoffs, and use this information to model preferences under more or less stringent assumptions of strategic choice. This involves modelling admission as a deterministic decision. However, they do not explicitly consider admissions uncertainty.

Agarwal and Somaini (2018) assume perfect knowledge of coarse student priorities, with uncertainty introduced by the actions of other decision-makers and the random tiebreaker. Rather than estimating atomic success probabilities for each school, Agarwal and Somaini (2018) estimate the full vector of assignment probabilities for a given ranked list, and therefore accommodate the rank-dependent priorities of the immediate-acceptance algorithm in use in Cambridge, Massachusetts. Admission probabilities are estimated by resampling and simulation of matchings. The relatively small cardinality of the set of ranked lists in their application means that this approach is computationally feasible. Calsamiglia, Fu, and Güell (2014) also estimates admission probabilities consisting of a deterministic priority plus a random tiebreaker.

In the current study, admission probabilities must be estimated with precise information about the tie-breaker (home-school distance) but little information about other aspects of the coarse priority structure. For example, we have no data about whether children have siblings already attending schools, or the details of families' religious worship. However, parents have perfect information about these important priority shifters, and this information will plausibly have a large effect on their choice behaviour. This sets up a selection-on-unobservables problem that must be addressed to obtain valid structural predictions of probability of admission. However, the fact that parents observe their own priority means that each family's rank order list potentially contains ample information to impute their priority status. This information is excludable from the structural admissions equation because each school does not observe the entire rank-order list, so if any characteristics of the entire rank-order list are correlated with probability of admission, this must be due to *priority-revelation* rather than a structural effect. Therefore, we can use parents' choice behaviour to make inferences about their likely priority level.

This idea of "self-revelation" is similar to the control-function approach developed for multinomial choice models by Dubin and McFadden (1984), and employed by Abdulkadiroğlu, Agarwal, and Pathak (2017) to estimate causal effects of schooling. It is also related to the matching models of Dale and Krueger (2002). *Priority-revealing* variables, excluded from the structural admissions equation, attenuate the omitted-variables bias and are then integrated out of the average structural function (ASF) (Blundell and Powell 2004). The latter is used for prediction of admissions probability on the second-stage dataset.

3.2 Identification of the Average Structural Function

Azevedo and Leshno (2016) develop a model of large many-to-one stable matching markets, such as school markets, and show that the equilibrium matching can be characterised by a set of market-clearing priority *cutoffs* at each school, which play a role analogous to market-clearing prices. Agarwal and Somaini (2018) build upon this to develop a general model of admission mechanisms which, in large markets, can be characterised by priority scores which may depend upon the decision-makers' reports, and market clearing cutoffs. They show that this model applies to a large class of stable and non-stable matching mechanisms.

Most admissions oversubscription policies in England share a common lexicographic structure. Two or more coarse priority groups define the high-level ranking, and a continuous measure breaks ties between children in the same priority group. The coarse priority structure can be represented by a function $\phi : \{0,1\}^K \to \mathbb{Z}_{\geq 0}$, that assigns to each child with binary criterion-vector $\mathbf{c}_i$ a priority group $\phi(\mathbf{c}_i) \in \{0, 1, \ldots, N_\phi\}$. The school fills its quota in order of priority from bottom to top, so that children with priority zero are allocated first. A continuous tie-breaker measure is used to assign priorities within each priority group. In the US and other countries it is common for a random lottery to be used to break ties. However, in England lotteries are rare and distance is the most commonly-used tie-breaker. West, Hind, and Pennell (2004) reported that 86% of secondary schools used distance as a tie-breaker. Since then the proportion is likely to have risen, since successive legislation has narrowed the range of legal priority rules. The tie-breaker can be incorporated into the ranking function by defining a monotonic mapping $\nu : [0, \infty) \to [0, 1)$. Then the full ranking function $r_j(\mathbf{c}_{ij}, d_{ij}) = \phi_j(\mathbf{c}_{ij}) + \nu(d_{ij})$.

However, we do not observe realisations of the coarse ranking function. Instead, we observe individual admission decisions $y_{ij}(\mathbf{c}_{ij}, d_{ij}) \in \{0,1\}$. Because of this, it is not possible to recover the entire coarse ranking function. However, observing y_{ij} allows for local identification of the index function and index function thresholds around the marginal priority group (i.e. the priority group for whom the distance tie-breaker is decisive). Figure 2 shows that observing repeated admission decisions at either side of the sharp discontinuity at the cutoff enables identification of the proportion of the population with criterion-vector $\mathbf{c}_{ij} = c$ belonging to the *marginal* coarse priority group $\mathcal{M}_j = \{i : \phi(\mathbf{c}_i) \in [\phi_l, \phi_{l+1})\}$, defined as the group for which the distance tie-breaker is decisive. This in turn identifies the parameters φ^* of the *local* index function and the upper and lower thresholds (ϕ_l, ϕ_{l+1}) of the local index function to scale and location for the marginal priority group. By extension the proportions belonging to the *infra-marginal* group $\mathcal{I}$, grouping all coarse priorities higher than the marginal group, and the *extra-marginal* group $\mathcal{E}$ grouping priorities lower than the marginal group, are also identified.

In the current application the criterion-vectors $\mathbf{c}_i$ are not observed. Instead, we observe a vector of proxies for priority $\mathbf{z}_{ij}$. This vector can be further split into pre-determined *priority-shifters* $\mathbf{z}_{1ij}$, and endogenous

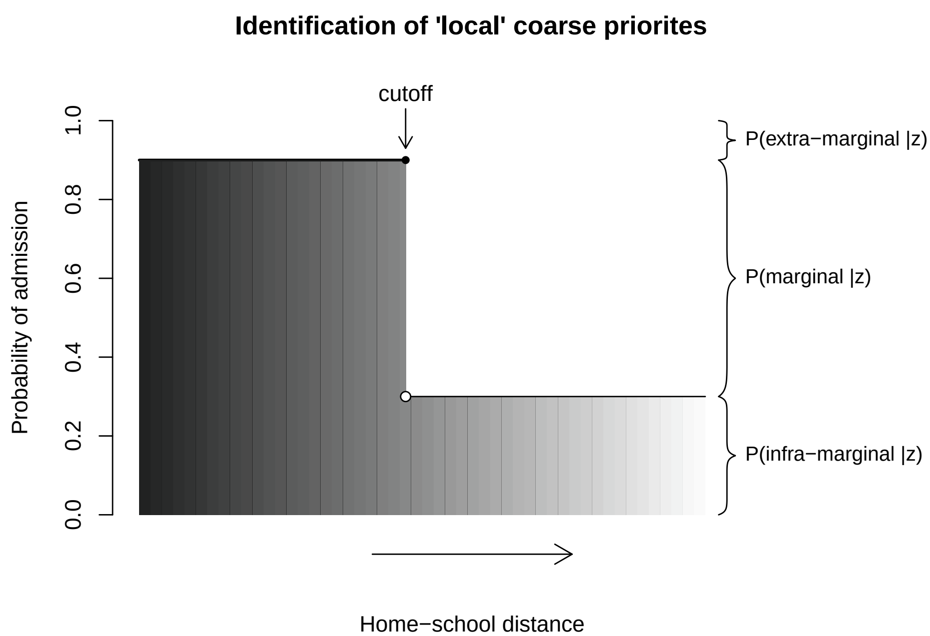

Figure 2: *Diagram of the structural mixture model that jointly estimates membership of the* marginal *coarse priority group, and the distance tie-breaker cutoff, for each school. If year-on-year variations in demand are not too large, the position of the distance cutoff will vary, but the proportions belonging to the three identified priority groups remains constant, and detailed knowledge of the predictors of other priority groups is not required.*

priority-revealing variables $\mathbf{z}_{2ij}$. An example of the first kind of variable is ethnicity, which may be correlated with religious worship and the likelihood of having older siblings at a school, and is pre-determined at the time of decision-making. An example of the second kind might be the size or composition of the full ranked list, besides the current school. For the current application, variables that are endogenous with respect to long-run strategies but pre-determined at the time of application, such as residential location, primary school choice and prior attainment, are treated as *priority shifters* and included in $\mathbf{z}_{1ij}$. Priority-shifters are included in the structural function, and priority-revealers are integrated out.

Although the full ranking function is not identified, the *local* ranking function (i.e. the ranking function that identifies the parameters predicting membership of the marginal priority group and its neighbours) is sufficient for predicting admission probabilities and counterfactual matchings as long as the year-on-year variation in the cutoff is not too large, that is, as long as the cutoff varies so that the marginal priority group remains the marginal priority group. This assumption can be checked by simulating allocations using resampled rank-order lists and the estimated rankings.

3.3 Empirical specification

The data is a set of one or more admission decisions $\mathbf{y}_i$ for each individual, the observable proxies for priority criteria $\mathbf{z}_i$ and the home-school distances $\mathbf{d}_i$. The likelihood is

$$P(y_{ij}|\boldsymbol{\varphi}, \Delta_j, z_{ij}, d_{ij}) = \begin{cases} P(i \in \mathcal{I}_j|\mathbf{z}_{ij}) + P(i \in \mathcal{M}_j|\mathbf{z}_{ij})\mathbf{1}\{d_{ij} < \Delta_j\} & \text{if } y_{ij} = 1 \\ P(i \in \mathcal{E}_j|\mathbf{z}_{ij}) + P(i \in \mathcal{M}_j|\mathbf{z}_{ij})\mathbf{1}\{d_{ij} \geq \Delta_j\} & \text{if } y_{ij} = 0. \end{cases}$$

The group-membership probabilities are estimated using an ordered logit threshold-crossing model with separate parameters for each school type, so that the full vector of parameters is $(\{\phi_{1t}, \phi_{2t}, \varphi_t\}_{t\in T}, \{\Delta_j\}_{j\in J})$ where (ϕ_{1t}, ϕ_{2t}) are upper and lower thresholds for the marginal priority group for school type t, φ_t are priority-model parameters, and Δ_j are school cutoffs for school j.

Variables in $\mathbf{z}_1$ include each student's primary school denomination (RC, C of E, other), two-way ethnicity (white, minority), Pupil-premium status, binned KS2 test score, and an indicator for whether the school is the student's nearest school, intended to proxy geographical priority areas. Variables in $\mathbf{z}_2$ include the size of the rank-order list, the position in the ranking of the current school, the number of grammar schools ranked (excluding the current school), and the number of faith schools ranked (excluding the current school).

The average structural function (Blundell and Powell 2004) may be used with the estimated parameters for imputation of ρ_{ij} into the second stage dataset. The average structural function is defined as

$$ASF(\boldsymbol{z}_1, d; \Delta) = \sum_{i=1}^{N} P(i \in \mathcal{I}_j|\boldsymbol{z}_1, \boldsymbol{z}_{2ij}) + P(i \in \mathcal{M}_j|\boldsymbol{z}_1, \boldsymbol{z}_{2ij})\mathbf{1}\{d < \Delta\}.$$

Integration over the population distribution can more easily be achieved by taking quantile means of $\boldsymbol{z}_2$ in the population, and redefining as a sum over quantiles q:

$$ASF(\boldsymbol{z}_1, d; \Delta) = \sum_{q=1}^{Q} P(i \in \mathcal{I}_j | \boldsymbol{z}_1, \bar{\boldsymbol{z}}_{2q}) + P(i \in \mathcal{M}_j | \boldsymbol{z}_1, \bar{\boldsymbol{z}}_{2q}) \mathbf{1}\{d < \Delta\}.$$

3.4 Estimating preferences under strategic choice

Modelling the rational selection of schools in a constrained rank-order list is a hard problem. Despite considerable recent attention devoted to the topic, the full model of expected utility maximisation remains intractable, even after imposing strong parametric assumptions. For this reason, this paper introduces a simple model, and characterises it as an approximation to the full model of expected utility maximisation under uncertainty. First, let us introduce the full model.

The canonical model of strategic school choice is the *downward recursive* portfolio choice model of Chade and Smith (2006). Under this model, the decision-maker chooses a portfolio of items in a sequential lottery to maximise her expected utility subject to either a non-negative cost per item, or a maximum quota of items in the lottery, or both. Lottery weights in each slot are composed from the product of the individual failure probabilities of items in previous slots, and the individual success probability of the item in the current slot. Adapting the notation of Agarwal and Somaini (2018), the lottery L_{R_i,ρ_i} therefore depends not only on individual success probabilities ρ_i, but also upon the ranking R_i.

In a school choice context, items are schools j, and we assume that the decision-maker i optimises her stated preference list, subject to a maximum number of slots S. Parents have private indirect utilities for all schools $\{v_{ij}\}_{j \in J}$ and choose the list that maximises expected utility

$$u_{R_i} = \rho_{ir_1} v_{ir_1} + (1 - \rho_{ir_1}) \rho_{ir_2} v_{ir_2} + \cdots + \prod_{r'=1}^{S-1} (1 - \rho_{ir'}) \rho_{ir_S} v_{ir_S} + \prod_{r'=1}^{S} (1 - \rho_{ir'}) u_{i0}^{S+1}.$$

$u_{R_i}^{S+1}$ is the expected utility of failing to be allocated to any school and being assigned to a school with spare capacity by the local authority.

As the name implies, the *downward recursive* portfolio problem admits a recursive definition. The expected utility of the last option, conditional upon being rejected from schools ranked higher, is $u_{R_i}^{S} = \rho_{ir_S} v_{ir_S} + (1 - \rho_{ir_S}) u_{i0}^{S+1}$. At each slot s the expected utility, conditional on being rejected from schools above is

$$u_{R_i}^{s} = \rho_{ir_s} v_{ir_s} + (1 - \rho_{ir_s}) u_{R_i}^{s+1}.$$

The expected utility of the entire list is therefore

$$u_{R_i}^{1} = u_{R_i} = \rho_{ir_1} v_{ir_1} + (1 - \rho_{ir_1}) u_{R_i}^{2}.$$

Note that the existence of a recursive definition of the problem does not imply that there is a recursive solution to the problem. The definition above implies that at each slot s the decision-maker conditions on both the set of schools above that slot and the set of schools below that slot. An exhaustive search of all possible ranked lists quickly becomes infeasible. For example, a London resident deciding which six s chools to list, if only considering schools in the same borough (typically about 15 schools) would have to consider $15!/(15-6)! \approx 3.6$ million possible lists. The empirical evidence suggests that parents do not limit themselves to only considering schools in the same borough.

In spite of this combinatorial complexity, Chade and Smith (2006) show that the decision-maker can select an optimal lottery sequentially in $O(N \times S)$ time by adding, at each stage, the item (school) that provides the largest marginal improvement in expected utility. However, the solution is not sequential in the sense of progressing, forwards or backwards, through the sequence of slots $s \in 1, \ldots S$. At each stage, an addition can be at the beginning, in the middle, or at the end of the existing list. This result is therefore of little use for constructing a structural empirical model, as the researcher does not know the order in which schools were added, and a model combining all possible paths would be of similar complexity to an exhaustive search[1].

To construct a probability model note that, at slot s, conditioning on $(r_1, \ldots, r_{s-1}, r_{s+1}, \ldots, r_S)$, a parent chooses a school r_s such that

$$r_s = \underset{j \in J \setminus R_{i,s-1}}{\arg\max}\ \rho_{ij}(v_{ij} - u_{i0}^{s+1})$$

where $R_{i,k}$ denotes $(r_1, \ldots, r_k)$ and $R_{i,0} = \varnothing$. The full likelihood can therefore be reconstructed by integrating over the distribution of "downstream" expected utilities u_{i0}^{s+1}, giving

$$\mathcal{L}(R_i) = \int \cdots \int \left\{ \prod_{s=1}^{S_i} P\left(\rho_{ir_s}(v_{ir_s} - u_{i0}^{s+1}) \geq \rho_{ij}(v_{ij} - u_{i0}^{s+1}),\ \ \forall\, j \in J \setminus R_{i,s-1}\right) \right\} f(u_{i0}^2, \ldots, u_{i0}^{S+1}) \mathrm{d}u_{i0}^2, \ldots, u_{i0}^{S+1}.$$

Agarwal and Somaini (2018) show that utilities are non-parametrically identified in the downward recursive model. However, the likelihood is intractable as it involves integration over the complex joint distribution of the random variables $\{u_{i0}^s\}_{s \in 2, \ldots, S+1}$.

However, this formulation suggests a simplifying approximation. Assume that, at any slot s, $u_{i0}^{s+1} << v_{ij}$ for all $j \in J \setminus R_{i,s-1}$. Then the downstream utility becomes unimportant, and the likelihood can be approximated by

$$\tilde{\mathcal{L}}(R_i) = \prod_{s=1}^{S_i} P\left(\rho_{ir_s} v_{ir_s} \geq \rho_{ij} v_{ij},\ \ \forall\, j \in J \setminus R_{i,s-1}\right).$$

This approximation will be accurate if the prospect of the administratively-assigned school is sufficiently bad, and it is sufficiently likely.

Specifying a Cobb-Douglas cardinalisation of indirect utility leads to a log-linear model for the approximate slot utility

$$\log u_{ij}^s \approx \alpha_{D(i)j} + \beta_{D(i)} \log d_{ij} + \delta_{D(i)s} \log \rho_{ij} + \epsilon_{ij}$$

[1]The Marginal Improvement Algorithm (MIA) is used in the welfare analysis to calculate the expected utility of the optimal list

where

$$\alpha_{D(i)j} = \sum_{k=1}^{K} \gamma_{D(i)k} x_{jk} + \xi_j,$$

preference parameters are indexed by pupil demographic group $D(i)$, and $\delta_{D(i)s}$ is a scaling parameter that represents subjective probability weighting and allows for different degrees of strategic choice. Only $\delta_{D(i)s}$ varies by slot. The other parameters are structural parameters that characterise the shape of indifference curves between aspects of school quality, and as such do not vary as the choice set changes along the rank-order list.

If ϵ_{ij} is distributed as a standard Extreme-Value Type I (Gumbel) variable, the approximate likelihood becomes a ranked logit

$$\tilde{\mathcal{L}}\left(R_i|\mathbf{x}, \boldsymbol{d}_i, \boldsymbol{\rho}_i, (\beta, \gamma, \delta)_{D(i)}\right) = \prod_{s=1}^{S_i} \frac{\exp\left(\alpha_{D(i)r_s} + \beta_{D(i)} \log d_{ir_s} + \delta_{D(i)s} \log \rho_{ir_s}\right)}{\sum_{j \in J \setminus R_{i,s-1}} \exp\left(\alpha_{D(i)j} + \beta_{D(i)} \log d_{ij} + \delta_{D(i)s} \log \rho_{ij}\right)}.$$

School covariates $\boldsymbol{x}$ include: the proportion of pupils achieving GCSE[2] grades at A* to C (AC5); the proportion of pupils eligible for Pupil Premium (a low-income proxy) in the school; the proportion of white pupils in the school; the school's most recent Ofsted inspection[3] grade (dummies for Outstanding and Good); and a separate dummy for each school type (Community, RC, C of E, Other faith, Grammar) excluding Academy as the reference level.

4 Estimating welfare counterfactuals

Current UK education policy relating to inequalities of access to good schools focusses on the geographical distribution of households and schools, and presupposes the existence of "cold spots": areas without good schools within a commutable distance (Department for Education 2016). The uneven distribution of good schools is one possible explanation for the observed differences in the proportion accepted into their first choice school. There are two additional competing explanations: first, differences in preferences and differences in strategy may mean that some demographics, ethnic minorities, for instance, are more likely to apply to popular schools, and less likely to use private information about chances of admission to avoid congested schools; second, the details of admission policies may lead to structural inequality in chances of admission.

It is possible that all three factors contribute to observed patterns of sorting and unequal headline admission statistics. It is also possible that, underlying the headline admission statistics, there is no actual difference in ordinal welfare between different demographic groups. For example, it may be the case that some families strategically edit their choice sets, so that their stated first choice is not actually their most preferred school, whereas other families truthfully report their preferences.

[2] A set of qualifications taken at the end of secondary schooling at age 16.

[3] The Office for Standards in Education carries out inspections of all schools at roughly three-year intervals, and awards headline grades: Outstanding; Good; Requires improvement; Special measures.

To examine the relative contribution of geography, admissions rules, and strategy/preferences, we compute expected welfare under three scenarios. We work with the expectations of log utilities rather than the expectations of Cobb-Douglas utilities, as the expectation of an exponentiated Extreme Value type I random variable is infinite. Log utilities are scaled by $\beta_{D(i)}$, so that log utilities are on the scale of log distance (in km). Levels of utility have no direct economic meaning, and cannot be compared between groups with different preference functions, as the level of utility is only defined up to a constant that depends on preferences. However, differences of log utilities can be compared, and are equivalent to log ratios of distance. For example, a log welfare difference of 0.69 implies that the welfare difference is equivalent to having to travel approximately twice as far. A population-averaged preference function is also estimated, which imposes uniform preferences on all demographic groups, and allows direct welfare comparisons to be made.

First, we can estimate the expected utility of families' choice sets, ignoring capacity constraints, as

$$W_A = \frac{1}{I}\sum_{i=1}^{I} E_\epsilon\left[\log \max_{j\in J} u_{ij} \middle| R_i, \boldsymbol{x}, \boldsymbol{d}_i, (\beta, \boldsymbol{\gamma})_{D(i)}\right] = \frac{1}{I}\sum_{i=1}^{N} E_\epsilon\left[\max_{j\in J} \log u_{ij} \middle| R_i, \boldsymbol{x}, \boldsymbol{d}_i, (\beta, \boldsymbol{\gamma})_{D(i)}\right].$$

This represents the average quality of the local school market, irrespective of whether the child can access it, and takes account of direct disutility from distance, but not the effect of distance on chances of admission. A single sample from the joint posterior distribution of $(\beta, \boldsymbol{\gamma})_{D(i)}$ is drawn for each child, so that the population distribution of welfare incorporates posterior parameter uncertainty.

The distribution of $\boldsymbol{\epsilon}_i$ is conditioned upon a family's rank-order list R_i in the following way: although, under constrained lists, it may no longer be optimal to truthfully report *all* of one's preferences, it is still a dominant strategy to include those schools which *are* included in the list, in order of true preference (Fack, Grenet, and He 2015). We assume that decision makers do not play a dominated strategy, and therefore the distribution of unobservables must be such that the preference order between schools in the submitted rank-order list is respected. Note that this does not imply anything about preference ordering of schools *not* included in the list. For this reason, the simple analytical expression $E_\epsilon[\max_{j\in J} \log u_{ij}] = \log \sum_{j\in J} \exp(\mu^u_{ij})$ cannot be used. Instead, we approximate the expectation by simulating draws from

$$\epsilon | \mu^u_{ir_1} + \epsilon_{ir_1} > \cdots > \mu^u_{ir_S} + \epsilon_{ir_S}$$

using a Gibbs sampler to sample from truncated EV1 distributions for schools included in R_i, and sampling from an unconstrained EV1 distribution otherwise.

The second welfare calculation characterises the welfare that a rational parent can achieve by submitting an optimal list, given the constrained rank-order lists and school admissions criteria, and is

$$W_B = \frac{1}{I}\sum_{i=1}^{I} E_\epsilon\left[\log \max_{R'\in\mathcal{R}} E_{L(R')}[u_i] \middle| \hat{\boldsymbol{\rho}}_i, R_i, \boldsymbol{x}, \boldsymbol{d}_i, (\beta, \boldsymbol{\gamma})_{D(i)}\right],$$

where

$$E_{L(R')}[u_i] = \hat{\rho}_{ir'_1} \exp(\mu_{ir'_1} + \epsilon_{ir'_1}) + \sum_{s=2}^{S}\left[\prod_{w=1}^{s-1}(1 - \hat{\rho}_{ir'_w})\right] \hat{\rho}_{ir'_s} \exp(\mu_{ir'_s} + \epsilon_{ir'_s})$$

for some rank-order list R'. $\hat{\boldsymbol{\rho}}_i$ are calculated using the average structural function and a single draw from the joint posterior distribution of $(\boldsymbol{\phi}, \boldsymbol{\varphi}, \boldsymbol{\Delta})$ for each child. The calculation of $\max_{R' \in \mathcal{R}} E_{L(R')}[u_i]$ uses the marginal improvement algorithm (Chade and Smith 2006) to calculate the optimal list, for each draw of ϵ.

Finally we define the expected average welfare of the lists that were actually submitted by parents

$$W_C = \frac{1}{I} \sum_{i=1}^{I} E_\epsilon \left[\log E_{L(R_i)}[u_i] \middle| \hat{\boldsymbol{\rho}}_i, R_i, \boldsymbol{x}, \boldsymbol{d}_i, (\beta, \boldsymbol{\gamma})_{D(i)} \right],$$

and the average expected welfare of the allocations that children received

$$W_D = \frac{1}{I} \sum_{i=1}^{I} E_\epsilon \left[\log u_{ij_i} \middle| R_i, \boldsymbol{x}_{j_i}, d_{ij_i}, (\beta, \boldsymbol{\gamma})_{D(i)} \right].$$

If admission probabilities are well-calibrated, $1/I \sum_{i=1}^{I} E_\epsilon[\log u_{ij_i}]$ converges to $1/I \sum_{i=1}^{I} E_\epsilon \left[E_{L(R_i)}[\log u_i] \right] \leq 1/I \sum_{i=1}^{I} E_\epsilon \left[\log E_{L(R_i)}[u_i] \right]$ so we should expect W_C to be greater than W_D, with the size of the difference proportional to $\mathrm{Var}_{L(R)}[u_i]$. For this reason, W_D is used for analysis as it does not rely on the estimated admission probabilities.

These welfare calculations, and the differences between them, represent the relative contributions of geography, preferences/strategies, and admissions constraints to average welfare for different demographic groups. W_A represents the welfare attributable to the geographic distribution of children and schools of different quality, before accounting for congestion. The difference between W_A and W_B represents the welfare loss attributable to capacity constraints, schools' admissions criteria and the design of the allocation mechanism (including constrained list length). The difference between W_B and W_D represents the welfare loss attributable to sub-optimal selection of rank-order lists. Care is needed in interpreting the gap between W_B and W_D, since the difference also captures, to some extent, the quality of model fit. If some demographic group has more heterogeneous preferences that are less strongly predicted based on observables, then the difference between W_B and W_D may appear large. However, the variance of the estimate should also be larger so that inferences should still be valid.

5 Parameter Estimates

Estimates of all parameters are tabulated in the appendices. Figure 3 shows estimates of the Willingness to Travel for academic performance (proportion achieving five or more A*–C at GCSE) at 2.5km. The WTT is calculated as

$$WTT_{D(i)}(d) = \frac{d\gamma_{D(i)}^{(AC5)}}{\beta_{D(i)}}.$$

The estimates show that, with other demographics fixed, families of higher-KS2 children are always willing to travel further for a school with higher test scores than those of lower-KS2 children. In addition, ethnic minority families are willing to travel further for improvements in test scores than white families. For white families, willingness to travel is lower for Pupil Premium-eligible children. However, ethnic minority families' preferences, although varying strongly by KS2 attainment, vary very little by Pupil Premium status.

Table 1 shows large and precise estimates of δ_s, the "strategy" parameter for families ranking slot s. We interpret estimates of δ with caution, noting that the parameter is more likely to absorb bias due to the approximate likelihood than other parameters. The magnitude of δ is sensitive to both the slot and the total number of schools ranked. δ is largest for those ranking only one school, and declines sharply as parents rank more schools. For a given size of rank-order list, the parameter appears largest for the first preference, but does not appear to decline after the second preference. There may be a slight "insurance school" effect, where the parent has a larger δ for the school ranked last (i.e. "1^{st} of 1", "2^{nd} of 2", and "3^{rd} of 3"), although in general parents appear to take most account of probabilities in choosing their top-ranked school. There is a small but precisely-estimated increase in δ for parents applying for schools in LA's which allow them to rank no more than three schools (short-list LA's) compared to those allowing four to six. The estimated mean difference is 0.109 (95% CI = $[0.076, 0.139]$).

Figure 4 splits the estimates for one rank-position by demographic group. Three patterns are clear: δ is

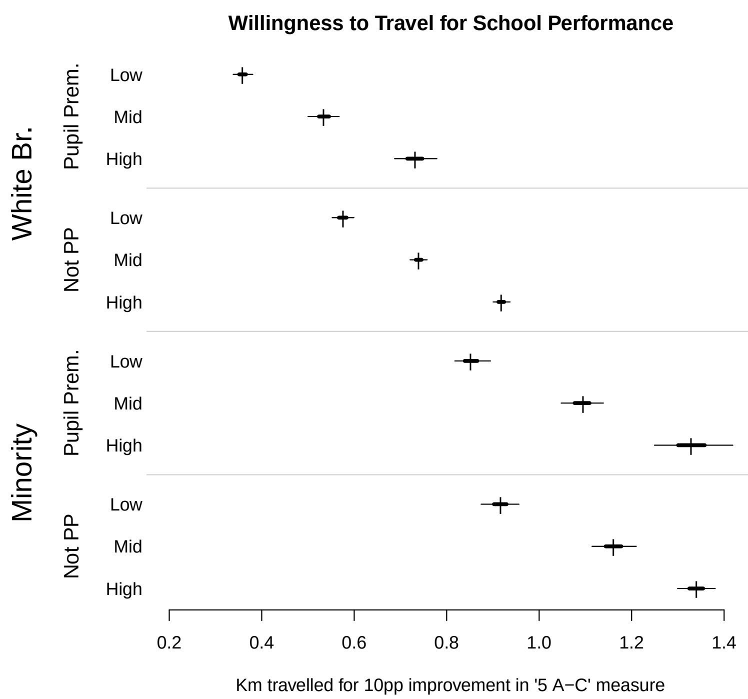

Figure 3: *Willingness to travel for improvement in a school performance indicator (% achieving 5+ A^*-C grades in GCSE exam), by pupil demographic group: two-way ethnicity; Pupil Premium status; and terciles of prior attainment at age 11 (Low/mid/High). Vertical line shows posterior mean; thick line shows posterior inter-quartile range; thin line shows 95% credible interval. The WTT is calculated at a fixed distance $d = 2.5km$ as $WTT = (d\gamma^{AC5}_{D(i)})/\beta_{D(i)}$*

smaller for families eligible for Pupil Premium; it is larger for white families; and it is larger for parents of children with higher Key Stage 2 (end of primary school) attainment than for parents of children with lower KS2 attainment. It might be concerning if the patterns in δ qualitatively mirrored the patterns in β, the distance parameter. This might suggest that the model is just capturing preferences for distance which is correlated with probability of admission, which might be a concern even though the first stage equation contains identifying variation. However, the pattern of δ by KS2 attainment is counter to the pattern of β by KS2 attainment **ref appendix**. The parents of higher-KS2 children appear to place more weight on admissions, but they place less weight on distance when choosing a school. Variation by demographic is not interacted with variation by rank-position or LA-type, so it is not possible to observe differences in δ across the full rank-order list by demographic group. The use of admission probabilities in decision-making appears to be dominated by white, non-Pupil Premium families, and particularly by more able children's families.

6 Welfare estimates

Figure 5 presents estimates of the national mean of welfare for each demographic group. These calculations are based on population-averaged estimates of the preference function, to allow between-group comparisons of welfare. W_A is the welfare of the choice set before accounting for admission constraints. This quantity

Table 1: Estimates of strategy parameter δ

	Rank-order slot					
	Long-list LA			Short-list LA		
Number ranked	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}
1	1.901			2.009		
	(0.022)			(0.027)		
2	1.359	1.278		1.468	1.385	
	(0.031)	(0.027)		(0.035)	(0.032)	
3	1.133	0.980	1.063	1.243	1.089	1.172
	(0.028)	(0.028)	(0.025)	(0.033)	(0.032)	(0.029)
4+	0.859	0.772	0.777			
	(0.027)	(0.029)	(0.026)			

Notes: The admission-probability scaling parameter δ is estimated by rank-order slot, by total number of preferences ranked, and for local authorities allowing 4-6 slots, versus those only allowing 3. It is also estimated for each demographic, but only estimates for white, medium-attainment non-Pupil Premium children are presented here. Estimates are posterior means, with posterior standard deviations in brackets.

is about 0.15 higher for ethnic minority families, which is accounted for by residential sorting into urban areas. In general, W_A is higher for families living in urban areas because the school market is more dense so disutility from travel is lower. Note that these results do not (yet) take account of variation in travel times. However, as the population is also more dense, these differences in utility are cancelled out by accounting for admissions (W_B).

Figure 6 presents the same welfare calculations for two predominantly-urban regions – London and the West Midlands – for comparison. The variation in W_A is much smaller, as the majority of families in these populations live in urban areas, but the variation in W_B is accentuated. The pattern of W_B in London echoes the pattern of δ shown in Figure 4; white, non-Pupil Premium families, and especially those with higher KS2 attainment, have higher admission-adjusted choice set utility than all other groups. The difference, for a high-KS2, non-Pupil Premium child is approximately 0.1, equivalent to a travel difference of 11%. There is no mechanistic reason why this should be the case, since the strategy parameter is not used in the welfare calculations. In the West Midlands, W_B for ethnic minority, Pupil Premium eligible families is around 0.3 lower than for white, non-Pupil Premium eligible families. This is equivalent to travelling about 35% further.

Welfare differences are further analysed in Figure 7. These show that nationally, the welfare loss due to

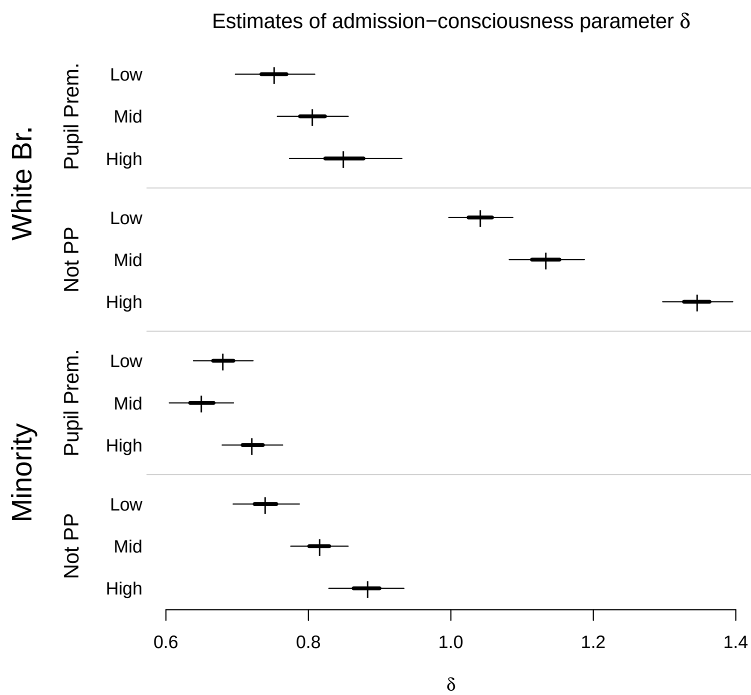

Figure 4: *Estimates of strategy parameter δ for slot $s = 1$ for parents ranking four or more schools, by pupil demographic group: ethnicity; Pupil Premium status; and terciles of prior attainment at age 11 (Low/mid/high). Vertical line shows posterior mean; thick line shows posterior inter-quartile range; thin line shows 95% credible interval.*

admission constraints, $W_A - W_B$, is a little larger than 0.2 for white families, and almost 0.4 for ethnic minority families, especially those eligible for Pupil Premium. The further welfare loss from not choosing the optimal list, $W_B - W_D$, is even larger, and the gap between white and ethnic minority families is also greater. Pupil Premium eligible, ethnic minority families have the largest loss in welfare due to sub-optimal choice, and non-Pupil Premium eligible, high-KS2, white families have the smallest loss in welfare due to sub-optimal choice. Figures 8 and 9 present these estimates for all English regions.

7 Conclusions and planned work

We have modelled parental preferences and school ranking functions with the intention of understanding the apparent inequality in outcomes of the secondary school admissions process. With the estimates, it will be possible to simulate parents' "true" rankings (i.e. without strategy) and explore whether the proportion of parents obtaining their first choice across different demographic groups is due to differential propensities to strategise, geography (selection by mortgage) or other aspects of the admissions system. In England, Weldon (2018) presents evidence that chances of admission to oversubscribed schools differ across demographic groups

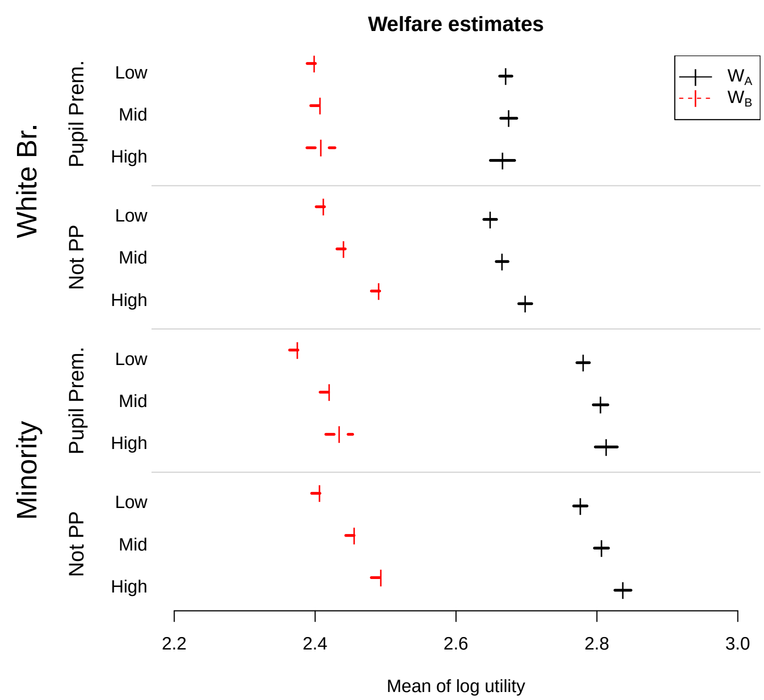

Figure 5: *Estimated average welfare, using a uniform preference function to allow comparisons between groups. W_A is the log utility of the choice set, before accounting for capacity constraints. W_B is the log utility of the choice set (i.e. the expected optimal utility that can be obtained), accounting for capacity constraints. Intervals are* $1.96 \times \frac{\hat{sd}}{\sqrt{I}}$.

after accounting for distance to the school, and that most of this difference can be accounted for by church schools' selection practices. However, that study does not explicitly account for strategy.

To be completed.

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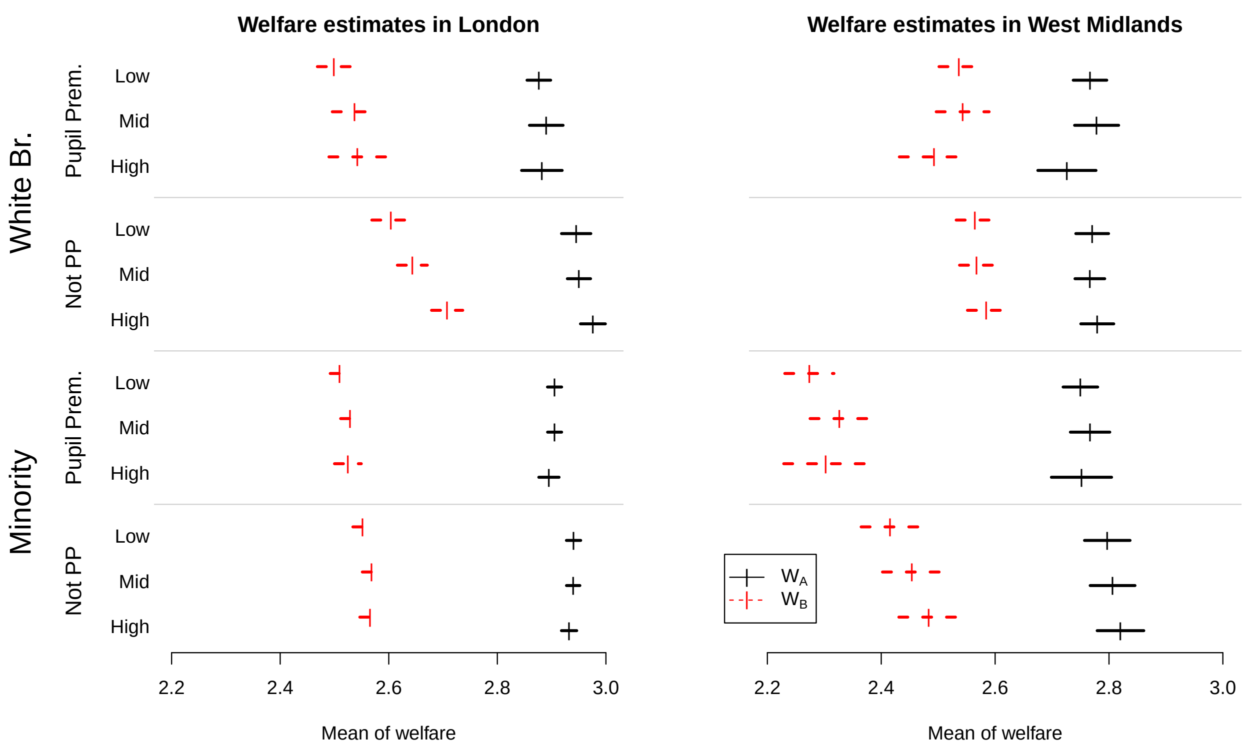

Figure 6: *Comparison of average welfare in two English regions: London and West Midlands (the other regions are plotted in the appendix). Both of these regions are predominantly urban and this controls for the urban/rural sorting of demographics that primarily affects estimates of W_A. In these plots the variation in W_A is attenuated, revealing variation in W_B. Intervals are $1.96 \times \frac{\hat{sd}}{\sqrt{I}}$.*

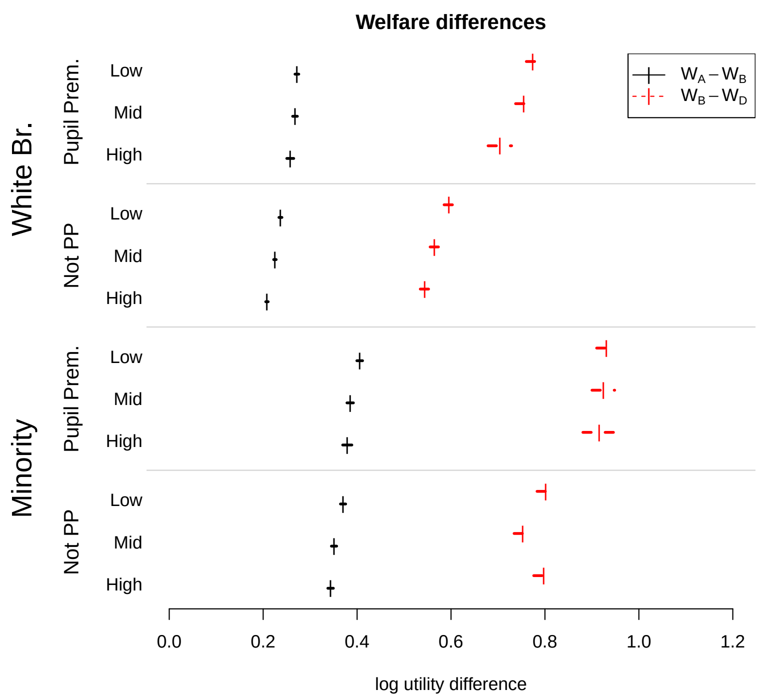

Figure 7: *Estimates of log-welfare differences: loss from capacity constraints –* $W_A - W_B$*; and loss from sub-optimal list choice –* $W_B - W_D$*. Differences are interpretable as log distance ratios. For example,* $W_A - W_B = 0.69$ *would imply a utility difference equivalent to having to travel roughly twice as far to school. Intervals are* $1.96 \times \frac{\hat{sd}}{\sqrt{I}}$.

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Table 2: Sample descriptives for the preference estimation

	N	%	N ranks	Avg dist. (km)	Avg % AC5*
All	84,991	100	175,383	3.65	62.4
Ethnicity					
White	57,940	68	112,291	3.69	61.4
Minority	27,051	32	63,092	3.57	64.6
Pupil Premium					
Not PP.	59,475	70	123,727	3.89	64.4
Pupil Prem.	25,516	30	51,656	3.09	58.0
KS2 attainment					
KS2: Low	31,448	37	62,090	3.21	58.6
KS2: Mid	30,217	36	62,486	3.65	62.6
KS2: High	23,326	27	50,807	4.24	67.4
Number ranked†					
One	27,287	32	27,287	2.84	59.8
Two	18,376	22	35,718	3.63	61.4
Three	21,722	26	62,903	3.88	62.1
Four	8,144	10	23,303	4.75	66.8
Five	3,664	4	9,631	3.73	65.0
Six	5,798	7	16,541	4.99	71.9
Region					
East	6,241	7	13,859	5.09	63.7
East Midlands	5,453	6	9,551	3.41	62.4
London	17,729	21	44,105	3.44	68.5
North East	4,503	5	8,586	2.59	56.9
North West	8,848	10	18,292	3.40	60.4
South East	12,591	15	27,318	5.06	66.0
South West	5,794	7	10,142	4.15	61.6
West Midlands	9,861	12	17,100	2.92	58.1
Yorkshire	13,894	16	26,319	2.80	57.5

Notes: The discrete choice model was estimated using a clustered choice-based sample, sampled from 35 clusters of 10 or 11 schools. * % of children achieving five or more A*–C grades at GCSE. † Only up to three ranked preferences were used in estimation for each person.

Table 3: % admitted to first-choice school in 2014

	England	London	Birmingham	Manchester
White	93	84	89	93
Black	73	68	72	77
S. Asian	75	71	68	80
Other	80	72	78	82
not FSM*	89	75	80	89
FSM	86	73	80	88
All	88	74	80	89

* Eligible for Free School Meals.

Table 4: % ranking each number of schools in 2014

# ranked	White	Black	S. Asian	Other	not FSM	FSM
London						
1	16	8	9	11	10	13
2-3	45	31	35	37	36	39
4-6	39	61	56	52	54	48
Birmingham						
1	32	19	17	25	23	30
2-3	51	43	37	44	47	43
4-6	18	37	46	30	30	28
Manchester						
1	38	25	28	34	33	40
2-3	54	60	57	52	56	53
4-6	8	14	15	14	11	7

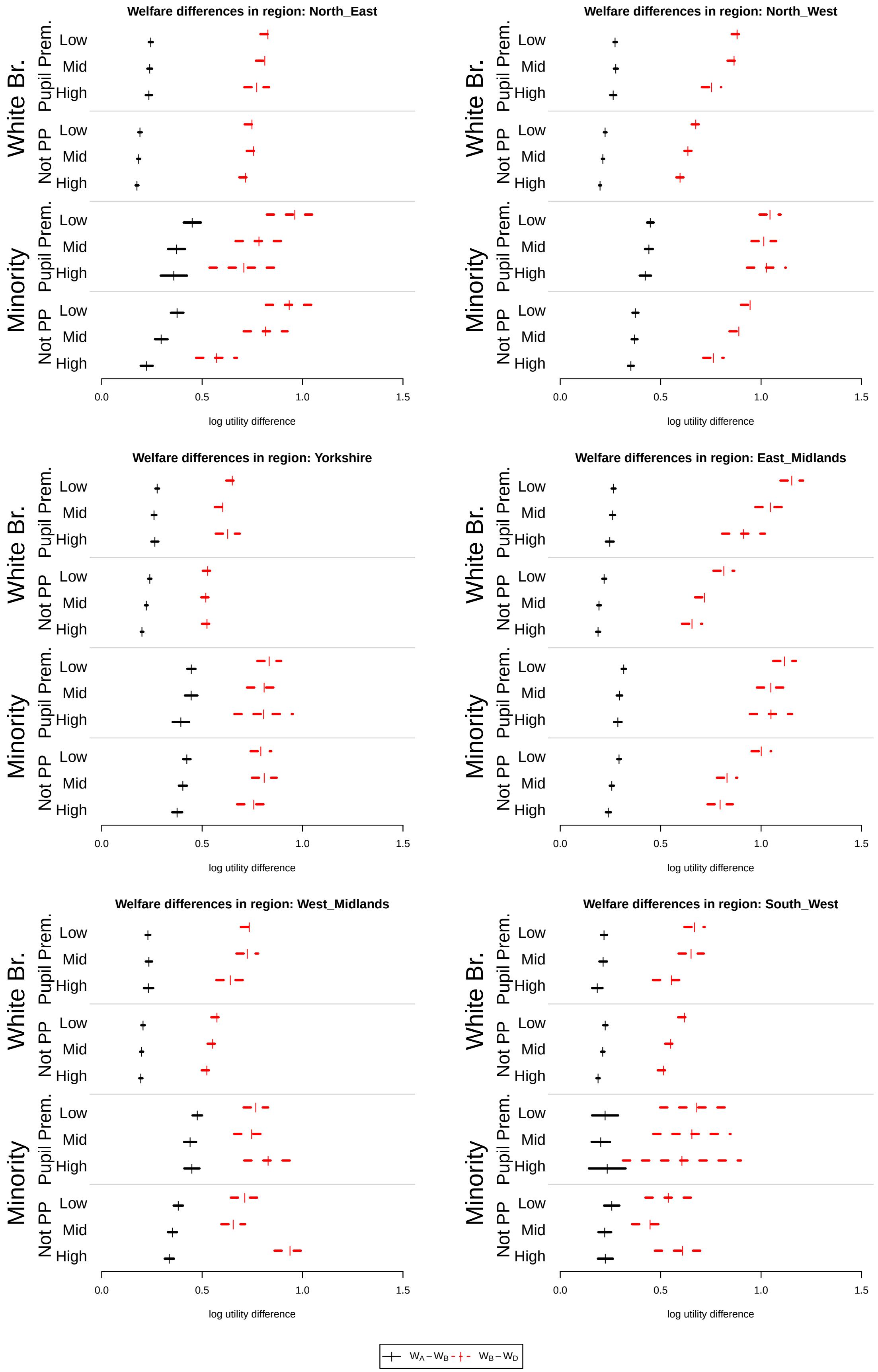

Figure 8: *Estimates of log-welfare differences: loss from capacity constraints –* $W_A - W_B$*; and loss from sub-optimal list choice –* $W_B - W_D$*. Differences are interpretable as log distance ratios. For example,* $W_A - W_B = 0.69$ *would imply a utility difference equivalent to having to travel roughly twice as far to school. Intervals are* $1.96 \times \hat{sd}$

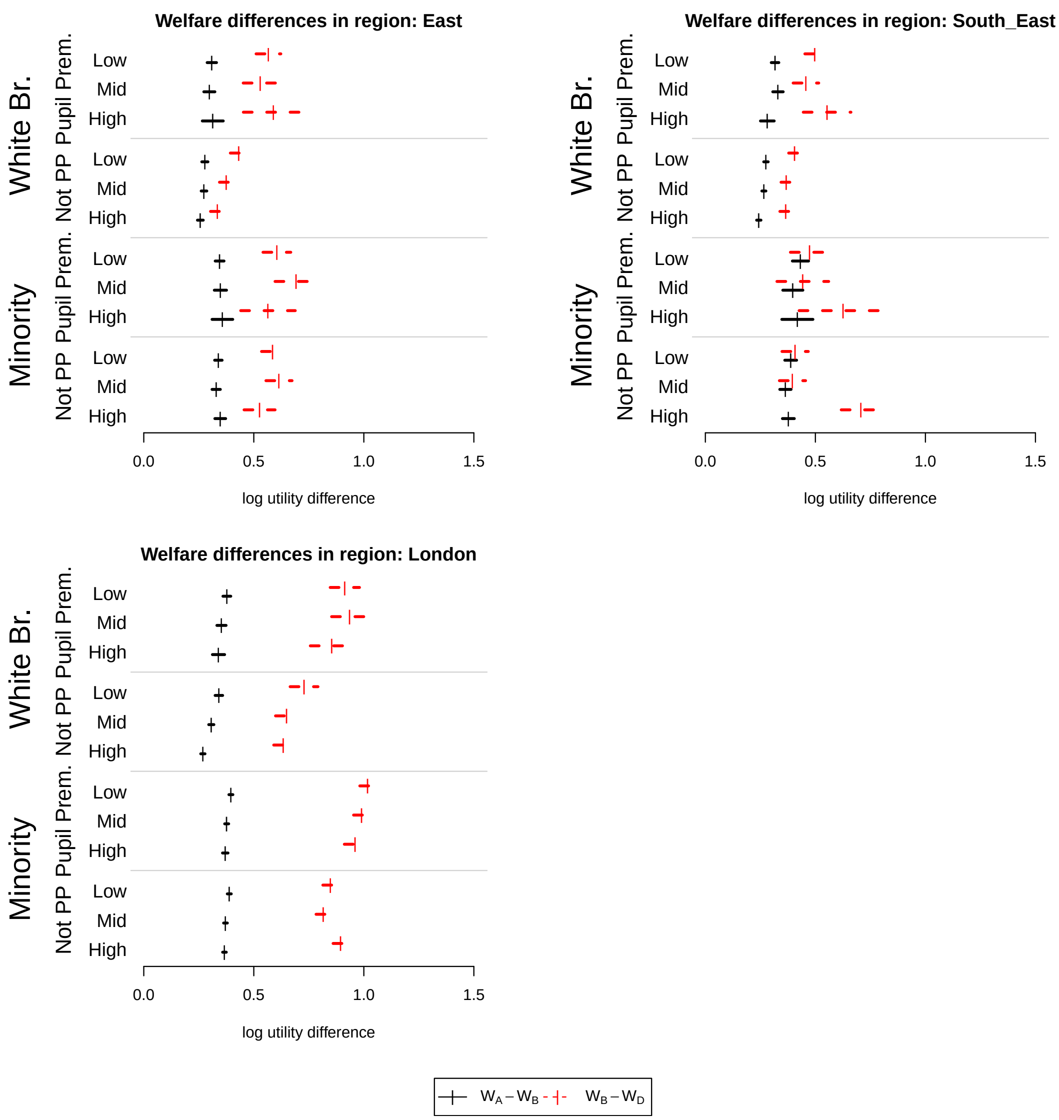

Figure 9: *Estimates of log-welfare differences: loss from capacity constraints – $W_A - W_B$; and loss from sub-optimal list choice – $W_B - W_D$. Differences are interpretable as log distance ratios. For example, $W_A - W_B = 0.69$ would imply a utility difference equivalent to having to travel roughly twice as far to school. Intervals are $1.96 \times \frac{\hat{sd}}{\sqrt{I}}$.*